ON THE STABILITY OF VISCOUS FLOW BETWEEN ROTATING CYLINDERS; FINITE GAP.

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ABSTRACT

A Mean Taylor Number is here introduced as a characteristic parameter governing stability of viscous flows between concentric rotating cylinders separated by a finite gap. This parameter is constructed on the basis of the forces causing convection and their mode of contribution to the instability. In particular for gaps equal to the radius of the inner cylinder considered by Chandrasekhar, it is found that the critical values of this Mean Taylor Number obey the laws governing the critical Taylor Number in the case of small gap.

INTRODUCTION

It was established[1] that the critical condition for convective instability of a horizontal layer of fluid with a vertical temperature gradient increasing with depth can be reduced to the critical conditions for convective instability of the same fluid layer, but with constant temperature gradient. The vertical temperature gradient concerning the latter case was determined from the former one by means of an appropriate averaging. A mathematical proof of this equivalence has so far been established only by invoking highly simplifying assumptions, and its generality may be limited. Numerically, however, high accuracy has justified its validity in various cases of temperature distributions.

Since the inversions in the ocean are in general accompanied by nonconstant gradients of temperature and salinity, the proposed principle of equivalence could provide a method for investigation of the stability of such inversions. Thus, investigation of the range of validity of this principle of equivalence is of importance. In order to direct the research concerning this range of validity, at the first stage other cases of more complicated distributions of convection causing forces have to be considered.

In this report an equivalent problem represented by a physically different model of stability of flow between rotating cylinders is considered. In the present case the nonconstancy of the convection causing forces is modeled by the variation of the angular velocity of the fluid. The special case of gap equal to the radius of the inner cylinder is considered in detail because of the available data of Chandrasekhar's[2] calculations concerning the critical conditions for instability in this case. However, due to singularity arising as a result of the change of sign of the angular velocity and the resulting more complicated physical interpretation of the results, only the case of cylinders rotating in the same direction is treated. The obtained result supports the validity of the mentioned principle of

equivalence for the considered cases of gap and angular velocity. Thus, it may be assumed that this principle gives in the general case an approximation which is good for practical purposes.

The above mentioned oceanographic analog of the problem treated here suggests that the principle of equivalence can be carried over in the investigation of stability of more complicated profiles of temperature and salinity than have hitherto been treated.

The subject of this paper is a segment of a sustained investigation initiated in 1957 concerning the effects of simultaneously impressed gradients in velocity, temperature, and salinity on stability of a class of fluid structures. This investigation was motivated by physical-mathematical considerations suggesting that simultaneously impressed velocity and temperature gradients on flows over curved boundaries can interact so as to be either mutually stabilizing or destabilizing(10). For this purpose, in Ref. (11), a model embodying only essential elements of this idea was treated. This model led to an interaction parameter consisting of Rayleigh-like and Taylor-like numbers. In subsequent investigations, including the present one, we have systematically relaxed simplifying assumptions of Ref. (1) and have extended the concept of gradient-interaction to include salinity gradients as well.

The Critical Conditions

The present calculations are concerned with a modification of Chandrasekhar's critical conditions(2) for stability.* Because of the simpler physical interpretation, only the case of

* Compare also the experimental verification of R. I. Donelly(3) and K. Kirchgaessner's(4) alternative treatment of the problem.

cylinders rotating in the same direction will be considered. The principle underling the calculations is the following:[1]

The considered problem of instability is one of balance between the centrifugal force and the pressure gradient, acting simultaneously on the same mass of fluid and damped by the viscosity. In the classical case of small gap and nearly equal angular velocities, the measure of this balance is given in terms of the Taylor Number

$$T = -4 A R_e^2 \omega$$

where (1)

$$R_e = \frac{d^2 \Omega_1}{\nu}, \quad \omega = A + \frac{B}{r^2}, \quad A = \frac{\eta^2 - \mu}{\eta^2 - 1}, \quad B = \eta^2 \frac{1-\mu}{1-\eta^2},$$

$$\eta = \frac{R_1}{R_2}, \quad d = R_2 - R_1, \quad \mu = \frac{\Omega_2}{\Omega_1}, \quad \eta \le r \le 1 .$$

In the general case the Taylor Number gives the measure of this balance only locally, as function of r. The radial perturbation velocity, directed as the above mentioned forces, will therefore be locally damped or amplified in proportion to the local value of the Taylor Number. For the integral effect, thus the Mean Taylor Number taken with weight function proportional to the radial perturbation velocity u'

$$\bar{T} = \frac{\int_0^h dz \int_0^{2\pi} d\varphi \int_\eta^1 r u' T \, dr}{\int_0^h dz \int_0^{2\pi} d\varphi \int_\eta^1 r u' \, dr}$$

or (2)

$$\bar{T} = -4 A R_e^2 (A + \gamma B), \quad \gamma = \frac{\int_\eta^1 \frac{u'}{r} dr}{\int_\eta^1 u' r \, dr},$$

will be characteristic. For the case $\eta = \frac{1}{2}$ the values of $\bar{T}_{cr}$ can be found by use of Chandrasekhar's results. The radial

perturbation velocity is given by Chandrasekhar's series

$$w = \sum_{i=1}^{\infty} P_i Q_i(\alpha_i z) \tag{3}$$

with coefficients given in Table I.

Table I. Coefficients of the series(3) for the state of marginal instability, $\eta = \frac{1}{2}$, according to Chandrasekhar(2).

μ	0·25	0·1667	0·1176	0·0
P_1	1	1	1	1
$-P_2 \times 10^2$	2·929	3·745	4·347	6·514
$-P_3 \times 10^2$	1·240	1·291	1·329	1·527

The eigenfunctions are Chandrasekhar-Reid's functions(5) satisfying four boundary conditions

$$Q_i(\alpha_i z) = A_i J_1(\alpha_i z) + B_i Y_1(\alpha_i z) + C_i I_1(\alpha_i z) + D_i K_1(\alpha_i z) \tag{4}$$

with eigevalues and coefficients given in the Table II.

Table II. Eigevalues and coefficients of the Bessel type functions satisfying four boundary conditions, $\eta = \frac{1}{2}$, according to Chandrasekhar-Reid(5).

i	α_i	A_i	B_i	C_i	D_i
1	9·499	1	-9·076	$1·006 \times 10^{-3}$	$4·396 \times 10^{2}$
2	15·74	1	-17·36	$3·676 \times 10^{-6}$	$-1·955 \times 10^{4}$
3	22·02	1	-22·47	$-8·864 \times 10^{-9}$	$5·912 \times 10^{5}$

The evaluation of the integrals involved the determination of γ as function of μ , the asymptotic series(6)

$$
\begin{aligned}
J_1(x) &\sim \sqrt{\frac{2}{\pi x}}\left\{\cos\left(x-\frac{3\pi}{4}\right)\left[1+\frac{0{\cdot}11719}{x^2}\right]-\sin\left(x-\frac{3\pi}{4}\right)\left[\frac{0{\cdot}375}{x}-\frac{0{\cdot}1025}{x^3}\right]\right\}\\
Y_1(x) &\sim \sqrt{\frac{2}{\pi x}}\left\{\sin\left(x-\frac{3\pi}{4}\right)\left[1+\frac{0{\cdot}11719}{x^2}\right]+\cos\left(x-\frac{3\pi}{4}\right)\left[\frac{0{\cdot}375}{x}-\frac{0{\cdot}1025}{x^3}\right]\right\}\\
I_1(x) &\sim \frac{e^x}{\sqrt{2\pi x}}\left[1-\frac{0{\cdot}375}{x}-\frac{0{\cdot}11719}{x^2}-\frac{0{\cdot}1025}{x^3}\right]\\
K_1(x) &\sim \sqrt{\frac{2}{\pi x}}\,e^{-x}\left[1+\frac{0{\cdot}375}{x}-\frac{0{\cdot}11719}{x^2}+\frac{0{\cdot}1025}{x^3}\right]
\end{aligned}
\tag{5}
$$

were used. By repeated integration by parts, for the integrals of the Bessel functions were obtained the asymptotic series

$$
\begin{aligned}
\int_{0{\cdot}5}^{1} J_1(\alpha_n z)\,z\,dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\Bigg[&\frac{\left|z^{1/2}\sin\beta_n(z)\right|_{0{\cdot}5}^{1}}{\alpha_n}-\frac{0{\cdot}3711}{\alpha_n^2}\left|z^{-1/2}\cos\beta_n(z)\right|_{0{\cdot}5}^{1}-\frac{0{\cdot}0683}{\alpha_n^3}\left|z^{-3/2}\sin\beta_n(z)\right|_{0{\cdot}5}^{1}-\\
&-\frac{1{\cdot}2461}{\alpha_n}\int_{0{\cdot}5}^{1}\frac{\sin\beta_n(z)}{z^{1/2}}\,dz\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\int_{0{\cdot}5}^{1} J_1(\alpha_n z)\,\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\Bigg[&-1{\cdot}422\left|z^{-1/2}\cos\beta_n(z)\right|_{0{\cdot}5}^{1}+\frac{0{\cdot}2891}{\alpha_n}\left|z^{-3/2}\sin\beta_n(z)\right|_{0{\cdot}5}^{1}-\frac{0{\cdot}0586}{\alpha_n^2}\left|z^{-5/2}\cos\beta_n(z)\right|_{0{\cdot}3}^{1}-\\
&-\frac{0{\cdot}0293}{\alpha_n^3}\left|z^{-7/2}\sin\beta_n(z)\right|_{0{\cdot}5}^{1}-1{\cdot}422\,\alpha_n\int_{0{\cdot}5}^{1}\frac{\sin\beta_n(z)}{z^{1/2}}\,dz\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\int_{0{\cdot}5}^{1} Y_1(\alpha_n z)\,z\,dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\Bigg[&-\frac{\left|z^{1/2}\cos\beta_n(z)\right|_{0{\cdot}5}^{1}}{\alpha_n}-\frac{0{\cdot}3711}{\alpha_n^2}\left|z^{-1/2}\sin\beta_n(z)\right|_{0{\cdot}5}^{1}+\frac{0{\cdot}0683}{\alpha_n^3}\left|z^{-3/2}\cos\beta_n(z)\right|_{0{\cdot}5}^{1}+\\
&+\frac{1{\cdot}2461}{\alpha_n}\int_{0{\cdot}5}^{1}\frac{\cos\beta_n(z)}{z^{1/2}}\,dz\Bigg]
\end{aligned}
$$

$$\int_{0.5}^{1} Y_1(\alpha_n z)\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\Big[-1.422\Big|z^{-1/2}\sin\beta_n(z)\Big|_{0.5}^{1} - \frac{0.2891}{\alpha_n}\Big|z^{-3/2}\cos\beta_n(z)\Big|_{0.5}^{1} - \frac{0.0586}{\alpha_n^2}\Big|z^{-5/2}\sin\beta_n(z)\Big|_{0.5}^{1}$$

$$+\frac{0.0293}{\alpha_n^3}\Big|z^{-7/2}\cos\beta_n(z)\Big|_{0.5}^{1} + 1.422\alpha_n\int_{0.5}^{1}\frac{\cos\beta_n(z)}{z}dz\Big]$$

$$\int_{0.5}^{1} I_1(\alpha_n z)\,z\,dz \sim \frac{1}{\sqrt{2\pi\alpha_n}}\Big[e^{\alpha_n}\Big(\frac{1}{\alpha_n}+\frac{0.3711}{\alpha_n^2}-\frac{0.0683}{\alpha_n^3}\Big) - e^{\alpha_n/2}\frac{0.7071}{\alpha_n} - \frac{0.62305}{\alpha_n^{3/2}}\int_{\sqrt{0.5\alpha_n}}^{\sqrt{\alpha_n}} e^{t^2}dt\Big]$$

$$\int_{0.5}^{1} I_1(\alpha_n z)\frac{dz}{z} \sim \frac{1}{\sqrt{2\pi\alpha_n}}\Big[e^{\alpha_n}\Big(-1.422+\frac{0.2891}{\alpha_n}+\frac{0.0586}{\alpha_n^2}+\frac{0.0586}{\alpha_n^3}\Big) + e^{\alpha_n/2}\Big(2.011-\frac{0.8163}{\alpha_n}\Big) + \frac{0.7110}{\alpha_n^{1/2}}\int_{\sqrt{0.5\alpha_n}}^{\sqrt{\alpha_n}} e^{t^2}dt\Big] \qquad (6)$$

$$\int_{0.5}^{1} K_1(\alpha_n z)\,z\,dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\Big[e^{-\alpha_n/2}\Big(\frac{0.7071}{\alpha_n}-\frac{0.5246}{\alpha_n^2}+\frac{0.1932}{\alpha_n^3}\Big) - \frac{1}{\alpha_n}e^{-\alpha_n} + \frac{0.62305}{\alpha_n^{3/2}}\int_{\sqrt{0.5\alpha_n}}^{\sqrt{\alpha_n}} e^{-t^2}dt\Big]$$

$$\int_{0.5}^{1} K_1(\alpha_n z)\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\Big[e^{-\alpha_n/2}\Big(2.011+\frac{0.8176}{\alpha_n}-\frac{0.3313}{\alpha_n^2}+\frac{0.3313}{\alpha_n^3}\Big) - e^{-\alpha_n}\Big(1.422+\frac{0.2891}{\alpha_n}\Big) - \frac{0.7110}{\alpha_n^{1/2}}\int_{\sqrt{0.5\alpha_n}}^{\sqrt{\alpha_n}} e^{-t^2}dt\Big]$$

where

$$\beta_n(z) = \alpha_n z - \frac{3\pi}{4}$$

By use of the asymptotic series(7)

$$\phi(z) = \int_0^{z^2}\frac{e^{-t}}{\sqrt{\pi t}}dt \sim 1 - \frac{e^{-z^2}}{\sqrt{\pi}\,z}\Big[1 - \frac{1}{2z^2} + \frac{1.3}{(2z^2)^2} - \frac{1.3.5}{(2z^2)^3} + \cdots\Big], \; -\frac{3\pi}{4} \le \arg z \le \frac{3\pi}{4}.$$

$$C(z) = \frac{1}{2\pi}\int_0^{z}\frac{\cos t}{\sqrt{t}}dt \sim \frac{1}{2} + \frac{\sin z}{\sqrt{2\pi z}}\Big[1 - \frac{1.3}{(2z)^2} + \frac{1.3.5.7}{(2z)^3} - \cdots\Big] - \frac{\cos z}{\sqrt{2\pi z}}\Big[\frac{1}{2z} - \frac{1.3.5}{(2z)^3} + \cdots\Big] \qquad (7)$$

$$S(z) = \frac{1}{2\pi}\int_0^{z}\frac{\sin t}{\sqrt{t}}dt \sim \frac{1}{2} - \frac{\cos z}{\sqrt{2\pi z}}\Big[1 - \frac{1.3}{(2z)^2} + \frac{1.3.5.7}{(2z)^3} - \cdots\Big] - \frac{\sin z}{\sqrt{2\pi z}}\Big[\frac{1}{2z} - \frac{1.3.5}{(2z)^3} + \cdots\Big]$$

the asymptotic expressions(6) are reduced to

$$\int_{0.5}^{1} J_1(\alpha_n z)\, z\, dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[\frac{1}{\alpha_n}\Big| z^{1/2}\sin\beta_n(z)\Big|_{0.5}^{1} + \frac{0.875}{\alpha_n^2}\Big| z^{-1/2}\cos\beta_n(z)\Big|_{0.5}^{1} + \frac{0.555}{\alpha_n^3}\Big| z^{-3/2}\sin\beta_n(z)\Big|_{0.5}^{1}\right]$$

$$\int_{0.5}^{1} J_1(\alpha_n z)\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[\frac{1}{\alpha_n}\Big| z^{-3/2}\sin\beta_n(z)\Big|_{0.5}^{1} - \frac{1.125}{\alpha_n^2}\Big| z^{-5/2}\cos\beta_n(z)\Big|_{0.5}^{1} - \frac{2.695}{\alpha_n^3}\Big| z^{-7/2}\sin\beta_n(z)\Big|_{0.5}^{1}\right]$$

$$\int_{0.5}^{1} Y_1(\alpha_n z)\, z\, dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[-\frac{1}{\alpha_n}\Big| z^{1/2}\cos\beta_n(z)\Big|_{0.5}^{1} + \frac{0.875}{\alpha_n^2}\Big| z^{-1/2}\sin\beta_n(z)\Big|_{0.5}^{1} - \frac{0.555}{\alpha_n^3}\Big| z^{-3/2}\cos\beta_n(z)\Big|_{0.5}^{1}\right]$$

$$\int_{0.5}^{1} Y_1(\alpha_n z)\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[-\frac{1}{\alpha_n}\Big| z^{-3/2}\cos\beta_n(z)\Big|_{0.5}^{1} - \frac{1.125}{\alpha_n^2}\Big| z^{-5/2}\sin\beta_n(z)\Big|_{0.5}^{1} + \frac{2.695}{\alpha_n^3}\Big| z^{-7/2}\cos\beta_n(z)\Big|_{0.5}^{1}\right]$$

$$\int_{0.5}^{1} I_1(\alpha_n z)\, z\, dz \sim \frac{1}{\sqrt{2\pi\alpha_n}}\left[\frac{e^{\alpha_n}}{\alpha_n}\left(1 - \frac{0.8749}{\alpha_n} - \frac{0.3496}{\alpha_n^2}\right) - \frac{0.7071}{\alpha_n} e^{\alpha_n/2}\right] \tag{8}$$

$$\int_{0.5}^{1} I_1(\alpha_n z)\frac{dz}{z} \sim \frac{1}{\sqrt{2\pi\alpha_n}}\left[\frac{e^{\alpha_n}}{\alpha_n}\left(1 + \frac{1.126}{\alpha_n} + \frac{2.695}{\alpha_n^2}\right) - \frac{2.827}{\alpha_n} e^{\alpha_n/2}\right]$$

$$\int_{0.5}^{1} K_1(\alpha_n z)\, z\, dz \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[\frac{e^{-\alpha_n/2}}{\alpha_n}\left(0.7071 + \frac{1.237}{\alpha_n} - \frac{1.568}{\alpha_n^2}\right) - \frac{e^{-\alpha_n}}{\alpha_n}\right]$$

$$\int_{0.5}^{1} K_1(\alpha_n z)\frac{dz}{z} \sim \sqrt{\frac{2}{\pi\alpha_n}}\left[\frac{e^{-\alpha_n/2}}{\alpha_n}\left(2.829 - \frac{6.364}{\alpha_n} + \frac{32.151}{\alpha_n^2}\right) - \frac{e^{-\alpha_n}}{\alpha_n}\right]$$

For the numerical evaluation of the integrals of the first eigenfunction the asymptotic series (6) has been used, the integrals in the right side being evaluated by use of calculated values of Fresnet integrals and the error function. For the integrals of the higher eigenfunctions the expressions (8) has been used. The obtained values of γ for the state of marginal

instability are given in the second row of Table III.

Table III. Critical values.

μ	0·25	0·1667	0·1176	0·0
γ	1·794	1·797	1·798	1·803
$\bar{T}_{cr} \times 10^3$	1·719	1·706	1·695	1·662

In the third column of the same table are given the values of $\bar{T}_{cr}$ obtained by means of eq. (2). It can be seen that they differ by less than 2% from these of the equation[8]

$$\tilde{T}_{cr} = 1708\left[1 - 7.61\times10^{-3}\left(\frac{1-\mu}{1+\mu}\right)^2\right], \quad \tilde{T} = -4AR_e^2\,\frac{1+\mu}{2},$$

valid for the case of small gap. Since also the critical value of the wave number obtained by Chandrasekhar, if related to the gap is equal to 3.1 as in the case of small gap, it can be concluded that the proposed method of averaging reduces the problem to this of small gap.

The mentioned deviation of less than 2% can be attributed to two sources; the 1% accuracy of Chandrasekhar's results and to an η depending term in eq. (9). Thus, for $\eta > \frac{1}{2}$ a much greater deviation from eq. (9) can be expected. For $\eta < \frac{1}{2}$ the following approximate method can be used. Since the radial perturbation velocity is in the general case unknown, the approximation by $\sin^2\frac{\pi}{1-\eta}(1-\tau)$ -a function satisfying the boundary conditions for u', can be used. Thus, we obtain for γ

the approximation

$$\gamma \doteq \frac{-\ln\eta - \cos\frac{2\pi}{1-\eta}\left[Ci\left(\frac{2\pi}{1-\eta}\right) - Ci\left(\frac{2\pi\eta}{1-\eta}\right)\right] - \sin\frac{2\pi}{1-\eta}\left[Si\left(\frac{2\pi}{1-\eta}\right) - Si\left(\frac{2\pi\eta}{1-\eta}\right)\right]}{\frac{1-\eta^2}{2}} \tag{10}$$

where

$$Ci(x) = \int_x^\infty \frac{\cos u}{u}\,du\,, \quad Si = \int_x^\infty \frac{\sin u}{u}\,du\,.$$

By use of the tabulated values of the integral cosine function the eq. (10) gives for $\eta = \frac{1}{2}$ the approximation $\gamma \doteq 1{\cdot}805$. Since the obtained by means of this approximation values of $\bar{T}_{cr}$ differ by less than 1% from the "exact" values given in Table III, it is concluded that for $\eta < \frac{1}{2}$ this approximation gives good results.

Conclusions

As mentioned in the introduction, the proposed method of averaging reduces the problem of critical conditions for convective instability due to non-constant gradients of temperature and concentration of diffusive substance to an equivalent problem but due to constant gradients. It is possible that this principle of equivalence has only an asymptotic character, however, the present calculations suggest that even for significantly non-linear gradients the approximation obtained by use of this principle is good.

In applications the unknown weighting function can be approximated by a function satisfying only its boundary conditions. The present calculations suggest that such an

approximation gives good results. However the above considerations are not directly applicable in cases in which some of the gradients changes sign; in such cases, in analogy with the case of counter-rotating cylinders[9], a more complicated dynamical model is to be considered.

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